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OPTIMIZATION OF STRUCTURES WITH PASSIVE
DAMPING AND ACTIVE CONTROLS

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Vibration suppression may be approached in three ways: by optimal distribution of structural mass, by viscoelastic damping treatments, and by active feedback control systems. Software design tools that address all three approaches are needed for SDI structures so that each approach can be exploited with minimal weight penalties. This report documents development in structural optimization under dynamic loads, both steady-state and transient. New methods for these problems are derived and demonstrated on small-scale structural models. The methods address minimization of structural mass subject to constraints on peak responses in either domain, together with frequency constraints and side constraints on member sizes. The methods are exercised with and without active control systems. A follow-on effort is proposed which expands the initial optimization capability and incorporates optimization of damping treatments, links to a viscoelastic materials database, and a database for finite element analysis models, design models, responses, and response sensitivities.

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List of Symbols

Boldface symbols are vectors or matrices

A	Area
b, h	Rectangular beam cross-section dimensions
$b(\mathbf{X})$	Behavior or response function
$b_{\min}, b_{\max}$	Behavior function limits
$\mathbf{B}$	Viscous damping matrix
$\mathbf{C}, \mathbf{D}, \mathbf{E}$	Intermediate matrices used in the Newmark beta method. See section 5.
$f(\mathbf{X})$	Function to be minimized in an optimization problem
g	Modal damping
$g_j(\mathbf{X})$	A constraint, satisfied when $g_j(\mathbf{X}) \leq 0$
K	Torsion constant
$\mathbf{K}$	Stiffness matrix
$\mathbf{K}'$	Stiffness matrix sensitivity with respect to an unspecified design variable
i	Imaginary unit
k, m	Modal stiffness and mass for a particular mode: $k = \mathbf{\Phi}^T\mathbf{K}\mathbf{\Phi}, m = \mathbf{\Phi}^T\mathbf{M}\mathbf{\Phi}$. Eigenvectors are usually normalized so that $m = 1$.
$\mathbf{k}, \mathbf{m}$	Diagonal matrices of modal stiffness and mass.
$\mathbf{P}$	Load vector
$p(t)$	Time variation of load
t	Time
$\mathbf{U}$	Displacement vector
$\mathbf{X}$	Vector of design variables
$\mathbf{X}^L, \mathbf{X}^U$	Side constraints
Δt	Time step
λ	Eigenvalue
$\mathbf{\Phi}$	Eigenvector
σ	Stress
ω_n	Natural frequency

1. Introduction

1.1 Motivation

Air Force and SDI strategic defense missions will place stringent design requirements on all structures used for mission support, especially those related to precision tracking and pointing. Although these structures will have requirements related to static performance under the environmental disturbances of the system, many structural systems will be particularly sensitive to the dynamic environment, and reducing this effect will be required for their success. These systems will also be subjected to weight restrictions.

Techniques that can reduce the dynamic motion of these sensitive structures fall into two categories: vibration isolation and vibration suppression. Each of these can be further divided into categories of active and passive. Vibration isolation means reducing the transmission of energy from the sources to the sensitive component, usually by reducing the transmissibility of the connecting structures. Vibration suppression reduces the dynamic motion that a component experiences by suppressing the motion either at the attachment points or other critical points. It involves changing the mass and stiffness characteristics of the structure to change its modal characteristics, increasing structural damping, or augmenting the structure with active control systems which improve its modal characteristics. Active isolation and suppression – which have important weight, cost and complexity penalties – will be employed for space-based systems. However, there is no substitute for efficiency and good basic design in the passive primary structure, whether or not it is augmented by active systems. The importance of dynamic response in system performance dictates that optimal structural design must be considered from the beginning of the design cycle.

The challenges summarized above call for accurate, reliable, and versatile design and analysis tools. These tools must not just address individual disciplines, but must be capable of application in integrated design situations. The Phase I effort reported here has addressed several technology issues whose successful resolution has laid the groundwork for development of a software package that integrates these newly developed capabilities with existing methods. The result will be a design tool which may be used to reduce the response of large, real-world structures to the effects of various vibration environments with minimal weight penalty.

1.2 Results Achieved

The proposal that preceded this effort listed several technical problems related to optimization which would have to be solved before a real-world optimization capability for space structures could be developed. We are pleased to report that the Phase I work was successful in investigating and developing techniques for all of these problems. The technical objectives set forth in the proposal are repeated below and discussed in detail in this report.

1. Develop a method for optimization under steady-state dynamic loads.
2. Develop a method for optimization under transient dynamic loads.
3. Study structural optimization under the influence of active control systems.
4. Couple existing optimization software for structural damping design with an existing viscoelastic material database system.
5. Develop a method of optimizing beam cross-sections based on cross-section shape variables.

The first two developments, optimization for steady-state and transient dynamic response, required development of new methodology, which is spelled out in some detail in the following sections. The methods were programmed and demonstrated on small academic structures, although the software could also be used for testing with large, realistic models. In Phase II, it is expected that optimization will be applied to problems such as minimizing settling time in slewing maneuvers or minimizing transmission of disturbances from energy sources. We at CSA are excited about the potential we see in these developments and are eager to begin demonstrating them on real-world problems.

An important consequence of the development of optimization methods for dynamic loads is the ability to perform optimization of structures in the presence of active control systems. Integration of the two systems in a single analysis pass represents a significant advance over traditional methods which essentially implied decoupling of control system dynamics from structural dynamics. This is important because of the intimate interaction between control system dynamics and structural dynamics in typical space structures. The developments reported here will offer the designer an optimization tool based on a correct coupled analysis. However, no optimization of control system parameters was attempted in the current effort.

The advance represented by these developments is shown symbolically in Figure 1. It builds on two previous developments: simultaneous structure-control system analysis, and structural optimization with simpler responses. It will be combined with these developments along with optimization of damping treatments in the proposed integrated package. Simultaneous structure-control system design is at present a research topic which is not proposed for inclusion in the new software.

The other two developments (database interaction and beam cross-section design) are somewhat loosely related aspects of the design process that will be

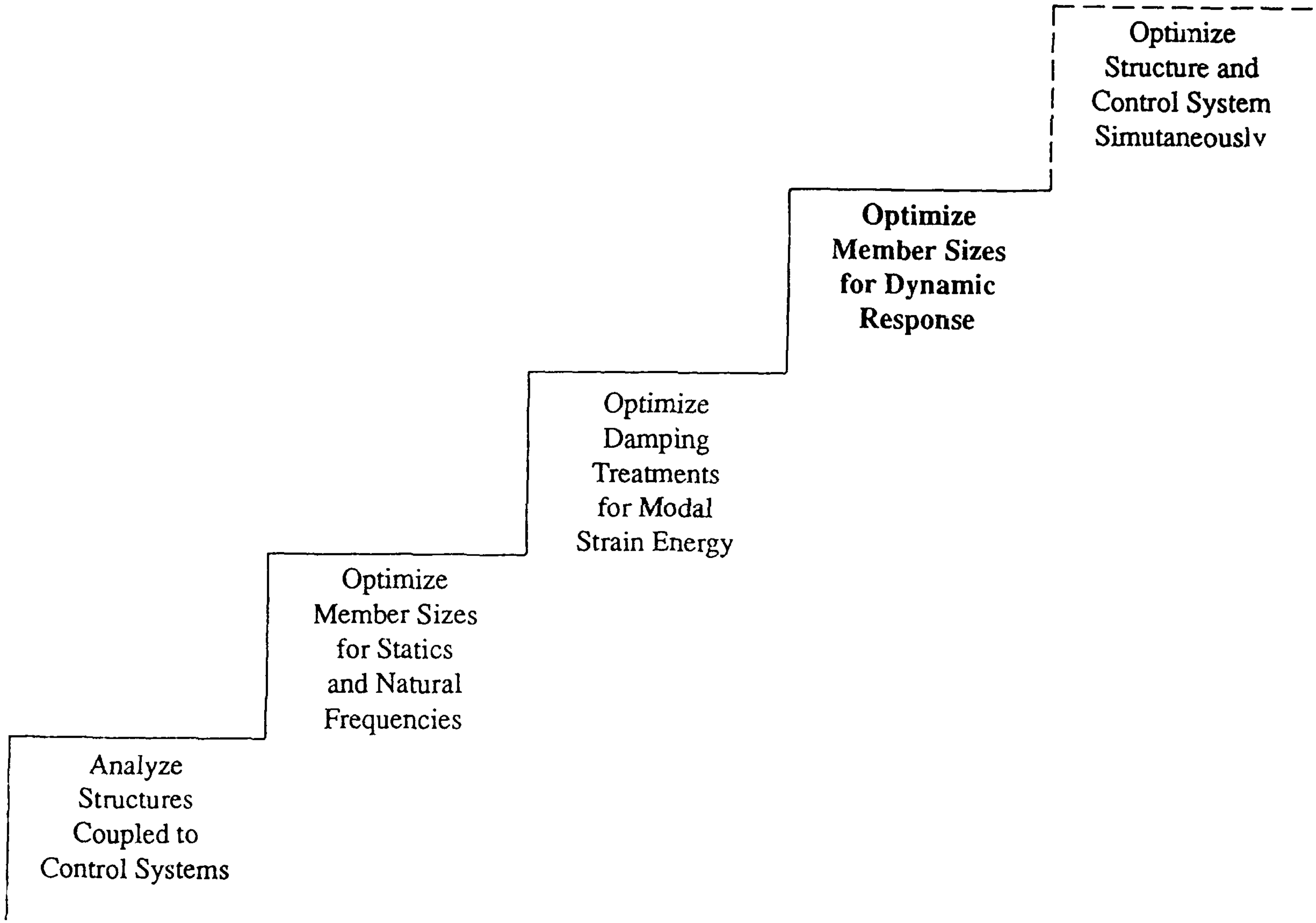

Figure 1. Advances in optimization technology for space structures

required in the Phase II implementation. Database interaction is needed in optimizing viscoelastic material selection so that the choices made by the optimizer can be translated into available materials. Beam cross-section design is necessary because design in terms of areas and moments of inertia, properties used for analysis, is unsatisfactory. For example, in the absence of other constraints, optimization software tends to make areas as small as possible and moments of inertia as large as possible. Also, even realistic areas and moments of inertia must be manually translated into cross-section details for final design.

A program called DYNOPT has been written to implement the methods that have been developed. The Fortran program runs on VAX computers and is linked to ADS, a general-purpose optimizer [1], and NASTRAN, a general-purpose finite element code. Additional code has been written in DMAP, NASTRAN's matrix manipulation language.

As will become apparent in the following sections, much has been accomplished. However, in keeping with the spirit of SBIR Phase I projects, no final products were developed. Software has been written and made to work, but much remains for Phase II. Separate special-purpose codes need to be integrated. Errors and "rough edges" need to be worked out, user experience and feedback accrued, and proper documentation provided. Large problems need to be undertaken as a demonstration to the SDI community. These matters are addressed further in Section 11.

2. Review of Structural Optimization

Structural optimization has been an active research field for about thirty years [2,3,4]. This activity is directed toward systematic methods for evolving structural designs that are optimal in the sense of some performance measure such as minimum weight, subject to constraints such as stress or displacement limits.

2.1 Basic Concepts in Optimization

An optimization problem is characterized by three components: design variables, constraints, and an objective function.

Design variables are aspects of the model that the user allows the optimization software to vary. In structural optimization, the design variables are typically sizing variables such as thicknesses, although configuration variables and even material properties are also possible design variables.

Constraints are classified as *side constraints* (simple upper and lower bounds on design variable values) and *behavior constraints* (response functions such as displacements that are generally implicit functions of the design variables). Side constraints are strictly enforced, but an initial design that violates one or more behavior constraints is permitted. Such a design is called *infeasible.* When an optimizer is started with an infeasible design, it ignores the objective function until it succeeds in satisfying all violated behavior constraints. In some applications, the sole purpose of the optimization problem is to find a feasible design.

The *objective function* is the function to be minimized. While weight is the typical objective in structural design, other criteria may be selected as well.

Stated formally, the problem is

$$\begin{array}{rll} \text{Minimize} & f(\mathbf{X}) & \\ \text{subject to} & g_j(\mathbf{X}) \leq 0 & j = 1, \ldots, m \\ \text{and} & \mathbf{X}_k^L \leq \mathbf{X}_k \leq \mathbf{X}_k^U & k = 1, \ldots, n \end{array} \tag{1}$$

where $\mathbf{X}$ is a vector of design variables, f is the chosen objective function, $\mathbf{X}_k^L$ and $\mathbf{X}_k^U$ are side constraints, and $g_j(\mathbf{X})$ are the behavior constraints. Behavior constraints are expressed in normalized form as

$$g_j = 1 - \frac{b(\mathbf{X})}{b_{\min}} \tag{2}$$

or

$$g_j = \frac{b(\mathbf{X})}{b_{\max}} - 1 \tag{3}$$

where $b(\mathbf{X})$ is some behavior corresponding to particular design variable values $\mathbf{X}$, $b_{\min}$ is a minimum acceptable value, and $b_{\max}$ is a maximum acceptable value.

2.2 Sensitivity Analysis

Optimization software uses search algorithms that rely on gradients or partial derivatives of the objective function $f(\mathbf{X})$ and the constraints g_j with respect to each design variable. For simple optimization problems, it is possible to approximate these gradients by finite differences. For structural analysis, finite difference methods would be prohibitive because they would require incrementing each design variable in turn and then performing a complete reanalysis. Thus the finite element software must be able to supply sensitivities, or gradients, along with the finite element results. For structural dynamic analysis, this means gradients of natural frequencies, mode shapes, steady-state responses, or transient responses.

Frequency sensitivities are a straight-forward matter [5]. Mode shape or eigenvector sensitivity algorithms have been worked out [6], but they are costly and could benefit from approximations similar to reanalysis techniques [7]. Such approximation techniques are being pursued by the present author under another Air Force contract and are expected to be available for exploitation in Phase II of the present effort.

2.3 Approximate Models

A key concept used in many structural optimization codes [8,9] is an *approximate model*. This concept, developed by Schmit and Miura [4], makes it possible to achieve near-optimal designs with very few complete finite element analyses. Finite element analyses with their accompanying sensitivity calculations consume the vast majority of the computer time in an optimization cycle. Thus, when properly applied, approximation techniques can achieve a great improvement in efficiency when compared with direct coupling of an optimizer with a finite element code. An outline flow diagram of optimization with an approximate model is shown in Figure 2.

The basic idea is to use the sensitivity information to set up a Taylor series expansion of both the objective function and the constraint function, i.e., they are linearized. Provided these functions are reasonably well behaved, the linearized functions form a reasonably good approximation over a reasonably wide range of design variable values. Thus, the optimizer can search for a local optimum within such a region. Since evaluation of the linearized functions is trivial, this local optimization process takes very little computer time. In fact, it is ironic that approximation techniques in many cases make questions of efficiency of the optimizer irrelevant. It makes no difference if the optimizer takes a lot of iterations, if the iterations cost practically nothing.

Only after the approximate optimization is complete is the structure re-analyzed. The linearization process is then repeated and the new approximate optimization problem is solved. At each such stage, move limits are imposed to insure that the structure is not changed so drastically that the linearization is not valid. The process of constraint linearization and optimization is repeated until no further design improvements can be found.

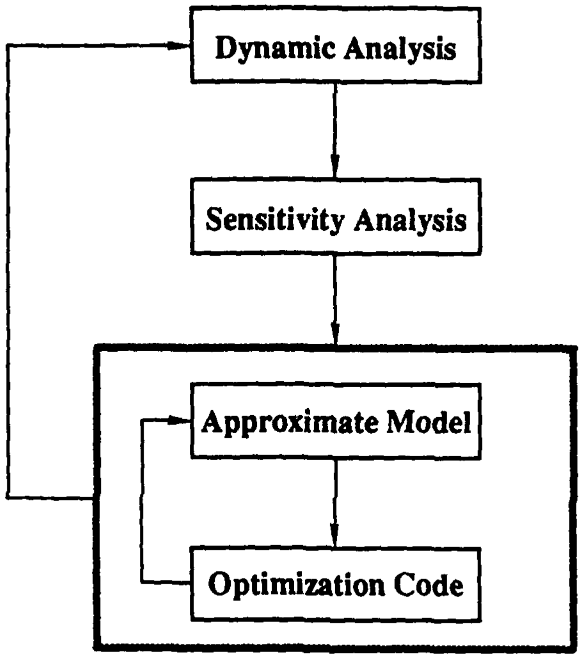

Figure 2. Two-level iteration with an approximate model

3. Review of Forced Dynamic Response Analysis

After discretization by finite elements, the equations of motion for forced response may be cast in the following general form:

$$\mathbf{KU} + \mathbf{B}\frac{\partial \mathbf{U}}{\partial t} + \mathbf{M}\frac{\partial^2 \mathbf{U}}{\partial t^2} = \mathbf{P}(t) \tag{4}$$

Where $\mathbf{K}$ is the stiffness matrix, $\mathbf{B}$ the damping matrix, $\mathbf{M}$ the mass matrix, $\mathbf{U}$ the node-point displacement vector, and $\mathbf{P}$ the applied load vector. An important special case arises when $\mathbf{P}(t)$ is sinusoidal, and transient effects are assumed to have died out. In this case the response is also sinusoidal and the equations may be written as a function of the excitation frequency ω.

$$[\mathbf{K}(1 + ig) + i\omega\mathbf{B} - \omega^2\mathbf{M}]\mathbf{U}(\omega) = \mathbf{P}(\omega) \tag{5}$$

Damping is difficult to characterize *a priori*. It is common practice to assume structural damping values that are uniform spatially but variable with frequency, i.e., for a particular mode i,

$$b_i = g_i\omega_i m_i \tag{6}$$

where g_i is obtained by interrogating a user-supplied structural damping table or function $g(\omega)$. Hence the $(1 + ig)$ term in (5).

3.1 Modal Superposition

Modal superposition is an efficient means of solving either the transient problem (4) or the steady-state problem (5). In this approach, a set of undamped natural frequencies and mode shapes $\mathbf{\Phi}$ are first calculated. Independent degrees of freedom are then transformed from node-point displacements $\mathbf{U}$ to a modal amplitude vector $\mathbf{q}$ using the mode shape matrix $\mathbf{\Phi}$.

$$\mathbf{U} = \mathbf{\Phi}\mathbf{q} \tag{7}$$

Substituting (7) into both (4) and (5) and premultiplying by $\mathbf{\Phi}^T$ yields transient and steady-state response equations expressed in modal coordinates:

$$\mathbf{kq} + \mathbf{b}\frac{\partial \mathbf{q}}{\partial t} + \mathbf{m}\frac{\partial^2 \mathbf{q}}{\partial t^2} = \mathbf{Q}(t) \tag{8}$$

and

$$[\mathbf{k}(1 + ig) + i\omega\mathbf{b} - \omega^2\mathbf{m}]\mathbf{q}(\omega) = \mathbf{Q}(\omega) \tag{9}$$

where $\mathbf{Q} = \mathbf{\Phi}^T\mathbf{P}$ is the modal load. $\mathbf{k}$ and $\mathbf{m}$ are diagonal matrices of modal stiffness, and mass, respectively. For the special case of uniform structural damping, the matrix $\mathbf{b}$ is also diagonal, so that the equations are uncoupled and thus easily solved. More general damping produces coupled modal damping matrices so that a small system of complex linear equations must be solved for each time step or frequency step.

4. Optimization with Steady-state Dynamic Loads

The challenge in minimizing the peaks in a steady-state frequency response is to compute the total derivative of a particular dynamic response peak with respect to design variables. Refer to Figure 3 which shows a hypothetical peak at some resonant frequency ω_o. If one were to calculate sensitivities at this particular frequency and then proceed with optimization, the results would almost certainly be unsatisfactory. While the optimizer might succeed in reducing the response at that particular frequency, it would likely do so by simply shifting the peak to a neighboring frequency. Thus it is necessary to compute the total derivative of the peak in a manner which accounts for its shift in frequency as well as its change in amplitude. Mathematically, we can express the total derivative of some constraint g_j with respect to a particular design variable $\mathbf{X}_k$:

$$\frac{dg_j}{d\mathbf{X}_k} = \left.\frac{\partial g_j}{\partial \mathbf{X}_k}\right|_{\omega=\omega_n} + \frac{\partial g_j}{\partial \omega}\frac{\partial \omega_n}{\partial \mathbf{X}_k} \tag{10}$$

The first term on the right-hand side represents the sensitivity of the response at the natural frequency ω_n, and the second term represents the shift in the natural frequency (at which we assume the peak occurs).

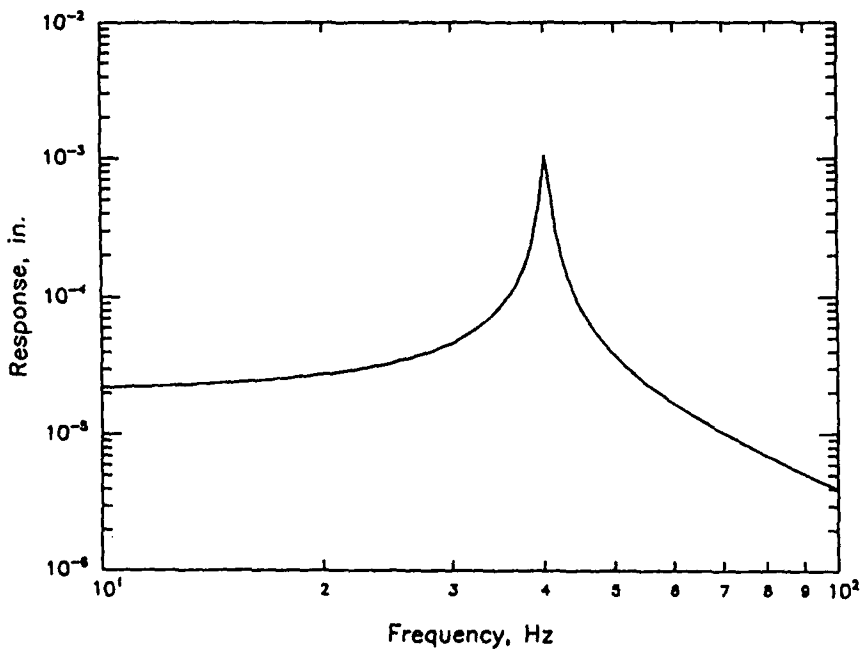

Figure 3. Hypothetical frequency response function

4.1 Optimization with Modal Structural Damping

The approach taken here rests on the assumption that all or nearly all of the response at a peak is due to a single normal mode. This would only be invalid in the case of closely space peaks, in which case one could minimize or constrain both peaks simultaneously and achieve the desired effect. We focus on a particular peak due to a mode with frequency ω_n and a mode shape $\mathbf{\Phi}$. The modal stiffness is the scalar value $k = \mathbf{\Phi}^T\mathbf{K}\mathbf{\Phi}$, (K being the stiffness matrix), the modal mass is the scalar value $m = \mathbf{\Phi}^T\mathbf{M}\mathbf{\Phi}$ (typically normalized to unit value), and the modal damping is g.

The peak displacement response at some point u_j due to this single mode may be expressed as

$$\mathbf{U}_j = \frac{\mathbf{\Phi}_j(\mathbf{\Phi}^T\mathbf{P})}{k(1+ig) - \omega^2 m} \tag{11}$$

where $\mathbf{\Phi}_j$ is the mode shape entry corresponding to displacement degree of freedom $\mathbf{U}_j$. For small damping values, the peak occurs very near the undamped natural frequency ω_n, for which $k - \omega_n^2 m = 0$, so that the magnitude of the peak complex response is simply

$$\mathbf{U}_j = \frac{\mathbf{\Phi}_j(\mathbf{\Phi}^T\mathbf{P})}{kg} \tag{12}$$

In this effort we are not attempting to maximize g, so we are left with three ways to minimize $\mathbf{U}_j$:

1. Reduce $\mathbf{\Phi}_j$. This means minimizing the participation of the mode in question at the particular response location. In a beam structure, for example, this might mean moving the nodes of the mode shapes toward the response point.
2. Reduce $\mathbf{\Phi}^T P$. This means reducing the modal participation of the load vector. In the case of a beam with a point load, again the node might be moved toward the load.
3. Increase k. Loosely speaking, this means adding stiffness to the structure in a manner that does not add mass, or adds it less than proportionately.

Using a general-purpose optimizer in conjunction with the peak response sensitivities derived above should, in theory, lead to optimized designs which take advantage of any of the three available avenues that were discussed above. The sensitivity data should contain all the information required to guide the design in whatever manner produces the most decrease in the objective function while satisfying the constraints. Note that we have not specified whether peak responses are to be used as the objective function or constraints. It is common practice to choose weight as the function to be minimized, while applying constraints to responses. For the rest of this discussion we will assume this choice. However, there is no reason why a response cannot be chosen as the objective function. This choice would have no effect on most of the required computations.

4.2 Sensitivity Analysis

Differentiating equation (12) using the chain rule and denoting derivatives with respect to design variables by primes:

$$\mathbf{U}_j' = \frac{\Phi_j'(\Phi^T P)}{kg} + \frac{\Phi_j(\Phi'^T P)}{kg} - \frac{\Phi_j(\Phi^T P)k'}{k^2 g} \tag{13}$$

The three terms on the right-hand side represent the three ways of reducing responses that were listed above. This sensitivity expression is seen to require sensitivities of the mode shapes Φ and the modal stiffness k. Of course, $\mathbf{U}_j$ could be reduced by increasing the modal damping factor g, but that is not the thrust of this effort. The modal stiffness sensitivity may be derived by differentiating $k = \Phi^T \mathbf{K} \Phi$:

$$k' = 2\Phi'^T \mathbf{K}\Phi + \Phi\mathbf{K}'\Phi \tag{14}$$

See the Appendix for derivations of frequency and mode shape sensitivities. The mode shape sensitivities are calculated using Nelson's method [6] as implemented in DMAP language by Wallerstein [10]. All these sensitivities depend ultimately on stiffness and mass matrix sensitivities which may be calculated either by a finite difference method or using the stiffness/mass sensitivity module in MSC/NASTRAN.

4.3 Optimization with Viscous Damping

Viscous damping forces are proportional to velocity and independent of frequency. The primary incentive for considering viscous damping in this effort is that velocity-sensitive active controllers are in effect viscous dampers. From a computational viewpoint, the main effect of viscous damping is to couple the equations of motion even when expressed in modal coordinates.

$$[\mathbf{k}(1+ig) + i\omega\mathbf{b} - \omega^2\mathbf{m}]\mathbf{q}(\omega) = \mathbf{Q}(\omega) \tag{15}$$

which can be solved directly:

$$\mathbf{q}(\omega) = [\mathbf{k}(1+ig) + i\omega\mathbf{b} - \omega^2\mathbf{m}]^{-1}\mathbf{Q}(\omega) \tag{16}$$

after which physical displacements may be obtained using (7).

5. Optimization with Transient Dynamic Loads

Transient dynamic motion is governed by the following equations of motion. These equations are given in matrix form, i.e., after spatial discretization but before temporal discretization.

$$\mathbf{KU} + \mathbf{B}\frac{\partial \mathbf{U}}{\partial t} + \mathbf{M}\frac{\partial^2 \mathbf{U}}{\partial t^2} = \mathbf{P}(t) \tag{17}$$

These equations are discretized in time by picking a suitable time step and applying a forward difference operator.

5.1 Newmark Beta Method

MSC/NASTRAN uses the Newmark Beta method of integration:

$$\left[\frac{\mathbf{M}}{\Delta t^2} + \frac{\mathbf{B}}{2\Delta t} + \frac{\mathbf{K}}{3}\right]\mathbf{U}_{n+2} =$$

$$\frac{1}{3}(\mathbf{P}_{n+2} + \mathbf{P}_{n+1} + \mathbf{P}_n) + \left[\frac{2\mathbf{M}}{\Delta t^2} - \frac{\mathbf{K}}{3}\right]\mathbf{U}_{n+1} + \left[-\frac{\mathbf{M}}{\Delta t^2} + \frac{\mathbf{B}}{2\Delta t} - \frac{\mathbf{K}}{3}\right]\mathbf{U}_n \tag{18}$$

Δt is the user-selected time step, and $\mathbf{M}, \mathbf{B}$, and $\mathbf{K}$ are mass, viscous damping, and stiffness matrices.

5.2 Sensitivity Analysis

As with steady-state loads, one can imagine beginning with the equations of motion, carrying out implicit differentiation with respect to design variables, and solving for the required sensitivity expression. Haftka [2] shows that at least one seeming difficulty can be dismissed: the time at which a peak occurs does not vary with design changes. To show this, consider a transient response constraint expressed as

$$g_j(\mathbf{X}, \mathbf{U}, t) \geq 0 \tag{19}$$

where $\mathbf{X}$ is a vector of design variables, $\mathbf{U}$ a vector of displacements, and t is time. Let t_p be the time at which a peak occurs and differentiate.

$$\left.\frac{dg_j}{d\mathbf{X}}\right|_{t=t_p} = \frac{\partial g_j}{\partial \mathbf{X}} + \frac{\partial g_j}{\partial \mathbf{U}}\frac{\partial \mathbf{U}}{\partial \mathbf{X}} + \frac{\partial g_j}{\partial t}\frac{\partial t_p}{\partial \mathbf{X}} \tag{20}$$

But since g_j is a local maximum at $t = t_p$ (and assuming t_p is not an end-point), we know that $\partial g_j/\partial t = 0$ so that the last term drops out and there is no dependence on the peak time.

We must still decide how to calculate the required sensitivity derivative. Write equation (18) as

$$\mathbf{D}\mathbf{U}_{n+2} = \frac{1}{3}(\mathbf{P}_{n+2} + \mathbf{P}_{n+1} + \mathbf{P}_n) + \mathbf{C}\mathbf{U}_{n+1} + \mathbf{E}\mathbf{U}_n \tag{21}$$

where

$$\mathbf{D} = \left[\frac{\mathbf{M}}{\Delta t^2} + \frac{\mathbf{B}}{2\Delta t} + \frac{\mathbf{K}}{3}\right] \tag{22}$$

$$\mathbf{C} = \left[\frac{2\mathbf{M}}{\Delta t^2} - \frac{\mathbf{K}}{3}\right] \tag{23}$$

$$\mathbf{E} = \left[-\frac{\mathbf{M}}{\Delta t^2} + \frac{\mathbf{B}}{2\Delta t} - \frac{\mathbf{K}}{3}\right] \tag{24}$$

If we assume the loads $\mathbf{P}$ are separable:

$$\mathbf{P} = p(t)\mathbf{P}_s(x, y, z) \tag{25}$$

where $\mathbf{P}_s$ is the spatial variation and $p(t)$ the time variation, we can differentiate (21) with respect to a design variable and get

$$\mathbf{D}'\mathbf{U}_{n+2} + \mathbf{D}\mathbf{U}'_{n+2} =$$

$$\frac{1}{3}(p_{n+2} + p_{n+1} + p_n)\mathbf{P}'_s + \mathbf{C}'\mathbf{U}_{n+1} + \mathbf{C}\mathbf{U}'_{n+1} + \mathbf{E}'\mathbf{U}_n + \mathbf{E}\mathbf{U}'_n \tag{26}$$

We want to solve for $\mathbf{U}'_{n+2}$ and we know everything else in the equation except $\mathbf{U}'_{n+1}$ and $\mathbf{U}'_n$.

$$\mathbf{U}_{n+2} = \mathbf{D}^{-1}\left[\frac{1}{3}(p_{n+2} + p_{n+1} + p_n)\mathbf{P}'_s\right.$$

$$\left. - \mathbf{D}'\mathbf{U}_{n+2} + \mathbf{C}\mathbf{U}_{n+1} + \mathbf{C}\mathbf{U}'_{n+1} + \mathbf{E}'\mathbf{U}_n + \mathbf{E}\mathbf{U}'_n\right] \tag{27}$$

Assuming zero initial conditions, $\mathbf{U}_0 = 0$, the sensitivities for the first three time steps can be written as

$$\mathbf{U}'_0 = \frac{1}{3}p_0[D]^{-1}\mathbf{P}'_s \tag{28}$$

$$\mathbf{U}'_1 = \mathbf{D}^{-1}\left[\frac{1}{3}(p_1 + p_0)\mathbf{P}'_s - \mathbf{D}'\mathbf{U}_1\right] \tag{29}$$

$$\mathbf{U}'_2 = \mathbf{D}^{-1}\left[\frac{1}{3}(p_2 + p_1 + p_0)\mathbf{P}'_s - \mathbf{D}'\mathbf{U}_2 + \mathbf{C}'\mathbf{U}_1 + \mathbf{C}\mathbf{U}'_1\right] \tag{30}$$

and the general formula is applied for subsequent time steps.

The above derivation was based on a "direct" approach, i.e., the physical stiffness, damping, and mass matrices $\mathbf{K}, \mathbf{B}, \mathbf{M}$ were used. However, the derivation applies equally well to a modal superposition approach. In modal superposition, the physical displacement degrees of freedom $\mathbf{U}$ are replaced by a vector of modal amplitudes each applying to one of the undamped normal modes in a chosen frequency range, i.e.,

$$\mathbf{q} = \mathbf{\Phi}^T \mathbf{U} \tag{31}$$

This is a common technique which is used because it provides a great increase in efficiency at the expense of a small error in the results.

In a modal formulation with eigenvectors normalized to unit modal mass, we have $m' = 0$, so that the constituent sensitivities reduce to $\mathbf{D}' = \mathbf{K}'/3$, $\mathbf{C}' = -\mathbf{K}'/3$, and $\mathbf{E}' = -\mathbf{K}'/3$, so that the general formula becomes

$$\mathbf{q}'_{n+2} =$$

$$\mathbf{D}^{-1}\left[\frac{1}{3}(p_{n+2} + p_{n+1} + p_n)\mathbf{Q}'_s - \frac{1}{3}\mathbf{K}'(\mathbf{q}_{n+2} + \mathbf{q}_{n+1} + \mathbf{q}_n) + \mathbf{C}\mathbf{q}'_{n+1} + \mathbf{E}\mathbf{q}'_n\right] \tag{32}$$

where the modal load is $\mathbf{Q} = \mathbf{\Phi}^T \mathbf{P}$. Converting back from modal to physical displacements requires another chain rule differentiation:

$$\mathbf{q}' = \mathbf{\Phi}'^T \mathbf{U} + \mathbf{\Phi}^T \mathbf{U}' \tag{33}$$

The load is normally not a function of any design variables, but in the case of a modal formulation, it involves the eigenvectors. Thus

$$\mathbf{Q}' = \mathbf{\Phi}'^T \mathbf{P} \tag{34}$$

At first glance this approach might seem inefficient since a complete forward integration of the sensitivity equations is required for each design variable. In practice, the integration proceeds very quickly when a modal formulation is used. However, one questions remains which has not been addressed. This concerns the number of modes that must be retained in the sensitivity solution for adequate accuracy. There can be no assurance that the number of modes adequate for satisfactory accuracy in the responses themselves would produce the same accuracy in the sensitivity equations. A countervailing consideration is the fact that accuracy in sensitivity is not as important as in the responses themselves. When approximate models are used, the same sensitivity values are used even after small design changes are made. Thus the inaccuracies inherent in approximate models would overshadow small errors in the sensitivity calculations.

6. Optimization in the Presence of an Active Control System

Active control systems are clearly needed in large flexible space structures. However, more conventional vibration suppression methods (passive damping methods and optimal structural design) are clearly preferable wherever they can be made to work because they require no input energy and generally weigh less than active control systems, relative to their effectiveness. This section discusses some optimization studies that were done in the presence of active control systems. The purpose of these studies is to demonstrate that active control systems and optimal structural design are complementary.

Control systems design is typically undertaken without much interaction with structural design. Structural designers and analysts compute natural frequencies and mode shapes of the uncontrolled structure and pass this information to control system analysts, who do their work using a body of knowledge and techniques known as *optimal control theory*. This procedure may not be suitable for space structures because of the close interaction between structural behavior and control system behavior. More to the point, traditional methods may not be suitable for evolution of designs that exploit passive vibration suppression methods wherever possible, and active control systems only where passive methods are inadequate.

The first step in integrating structural design with control system design is combined analysis. This means solving a set of equations which represents both the structural dynamics and the control system dynamics. NASTRAN has a provision for addition of control system variables which are governed by user-specified transfer functions and controller gains. With this capability, the designer is at least able to get an accurate assessment of the combined effects of active and passive damping. The NASTRAN capability translates the controller dynamics into terms which are added to the system stiffness, damping, and mass matrices. Controllers that respond to velocity sensors naturally contribute to the damping matrices. This is the reason why the alternate methoa of optimization with viscous damping was presented in Section 8. Other types of controllers have no effect on the optimization method, although the results may of course be much different in the presence of controllers.

The second step would be integration of control system design and structural design. This would involve simultaneous variation of structural and control system variables using integrated sensitivity equations. Some preliminary work has been done in this area [11,12] but no such development has been attempted here.

An example problem showing optimization in the presence of a control system is shown in Section 8.5.

7. DYNOPT Software

This section describes the software that was developed under this contract. The software consists of Fortran code and DMAP code which are described here.

7.1 Sequence of Operations

Following are the steps involved in running an optimization problem with DYNOPT:

1. Create a finite element model of the structure in the usual manner. In NASTRAN, element properties are not assigned directly to individual elements, but are arranged in property groups. Each group may be referenced by one or more elements. Design variables in DYNOPT refer to property groups, and thus property groups must be laid out with the optimization problem in mind.
2. Create a "design model," a specification of design variables, a particular displacement degree of freedom to be minimized, and other constraints which may concern weight, natural frequencies, etc. Currently, only ROD areas, QUAD4 thicknesses, and TRIA3 thicknesses are supported as design variables, but extension to other design variable types would not be difficult.
3. Calculate undamped normal modes.
4. Specify steady-state or transient dynamic loads.
5. Compute natural frequency and mode shape sensitivities in MSC/NASTRAN. These are used to compute derivatives of the selected peak displacements with respect to the user-supplied design variables.
6. Using an approximate model, perform a cycle of design optimization. Recalculate normal modes, dynamic responses, and sensitivities. At this point the user may review results, make changes, or proceed with another cycle of design optimization.

7.2 Fortran Code

The Fortran portion of DYNOPT is set up to do the following:

1. Handle user interface through screen management software. Actions are controlled through menu picks. There is a general screen, a screen for design variable display, one for natural frequency display, one each for frequency response and transient response displays, and one for submitting NASTRAN batch runs.

2. Read the analysis model from a user file to determine starting values of design variables, loads (including spatial, frequency, and time variation), and parameters.
3. Read the design model, a file prepared in a format similar to NASTRAN's bulk data. This file specifies design variables, constraints, and miscellaneous information.
4. Read the control file which consists of NASTRAN executive control and case control decks.
5. Prepare and submit a normal modes run for NASTRAN. The proper DMAP alters are inserted automatically in the executive control.
6. Prepare and submit a short NASTRAN DMAP run to generate load vectors.
7. Prepare and submit a NASTRAN DMAP sensitivity run.
8. Retrieve results from NASTRAN runs (eigenvalues, eigenvectors, modal stiffnesses, sensitivities, and loads).
9. Compute frequency response functions.
10. Integrate the time-dependent equations of motion.
11. Write frequency or time responses to files suitable for plotting.
12. Set up the approximate model and call ADS, the optimizer.
13. Maintain a history file showing design changes, response changes, and weight changes.

7.3 DMAP Code

The second part of DYNOPT is DMAP code. DMAP is NASTRAN's Direct Matrix Abstraction Program which allows various operations on matrices, vectors, and scalars, along with input and output of data on binary files. There is one DMAP modifications which is applied to Solution 63 and two stand-alone DMAP programs:

1. Alter for Solution 63: Code to write out weight, natural frequencies, modal stiffnesses, and mode shapes.
2. Stand-alone code to process load bulk data cards and write out binary load vectors.
3. Stand-alone code to compute sensitivities. This consists of five sections as follows:

(a) Compute stiffness and mass matrix sensitivities for the selected design variables. Code is in place to handle the following classes of design variables: element property changes, material property changes, and grid point moves. Only the first class is currently supported by DYNOPT.

(b) Compute weight sensitivities.

(c) Compute eigenvalue sensitivities.

(d) Compute eigenvector sensitivities.

(e) Compute modal stiffness sensitivities.

8. DYNOPT Example Problems

This section shows various sample problems that were chosen to illustrate the capabilities of DYNOPT. The primary intent of these examples was to check out the software and to demonstrate the basic capabilities. As such, it was necessary to keep them simple. More complex structures with multiple load paths and more complicated mode shapes would bring out the advantages of optimization more clearly. First, the difficulty in making manual design changes would be apparent. Second, the payoffs in weight versus performance would be more dramatic because of the multiple opportunities available for improvements in complex structures.

8.1 Box Beam with Steady-state Loads

The first problem was really intended to check out the software, but it also served to illustrate optimization by stiffening. The problem is a box beam as shown in Figure 4. Four design variables were chosen:

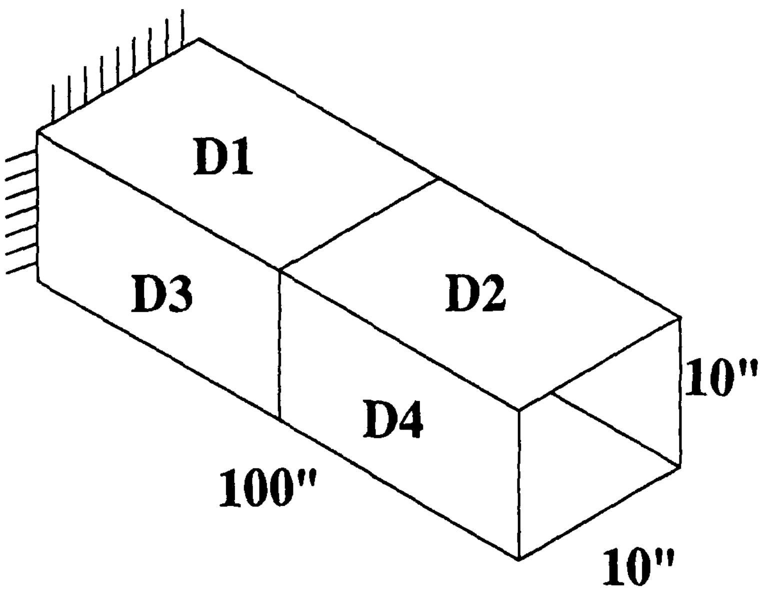

Figure 4. Box beam

1. D_1: Top and bottom thicknesses toward the root of the beam.
2. D_2: Top and bottom thicknesses toward the tip of the beam.
3. D_3: Side thicknesses toward the root of the beam.
4. D_4: Side thicknesses toward the tip of the beam.

A unit vertical tip load was imposed. The constraints chosen were:

1. Fundamental frequency to lie between 35 and 50 Hz.
2. Second frequency to lie between 40 and 70 Hz.
3. Peak tip response in mode 1 to be less than .001 inches.

The problem was started with all thicknesses at 2.0 inches. For this design, the response constraint was violated. The progress of the design is shown in Figures 5 through 8.

In the first three cycles, we see the weight increase slightly while the peak response decreases. During these iterations the optimizer does not consider the weight but only tries to satisfy the violated constraint. Thereafter the weight decreases slightly while the response constraint is satisfied (within close tolerance), and the second natural frequency is near its upper limit.

This problem, while academic, did serve to validate the software and to demonstrate that the approximation concept outlined in Section 2.3 provides convergence in only a few design cycles. Each design cycle entails a complete finite element analysis and sensitivity analysis.

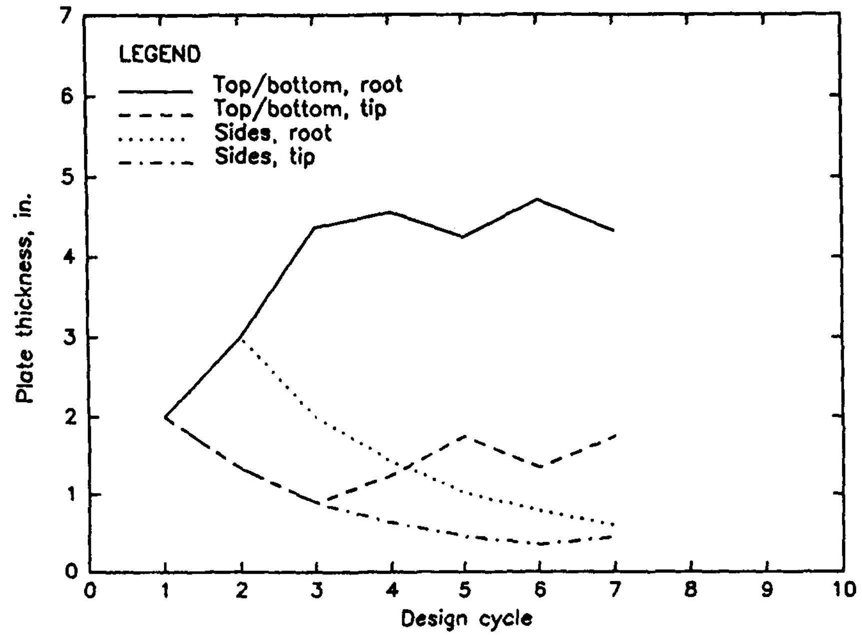

Figure 5. Box beam design variable history

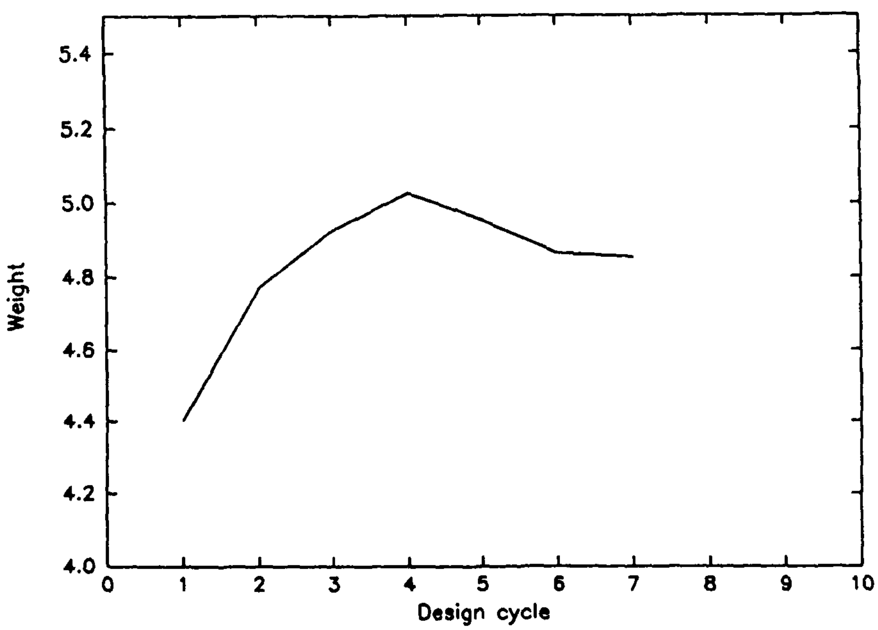

Figure 6. Box beam weight history

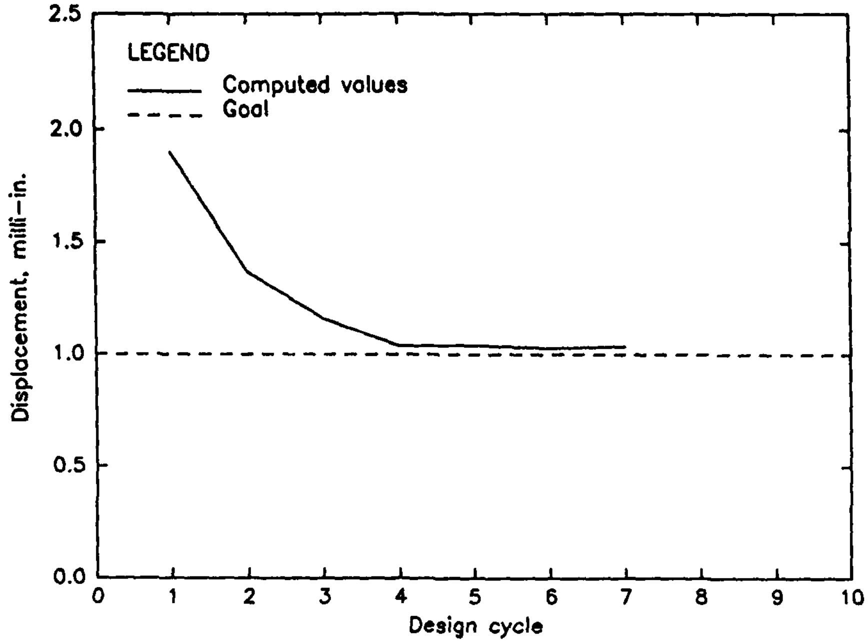

Figure 7. Box beam first mode peak history

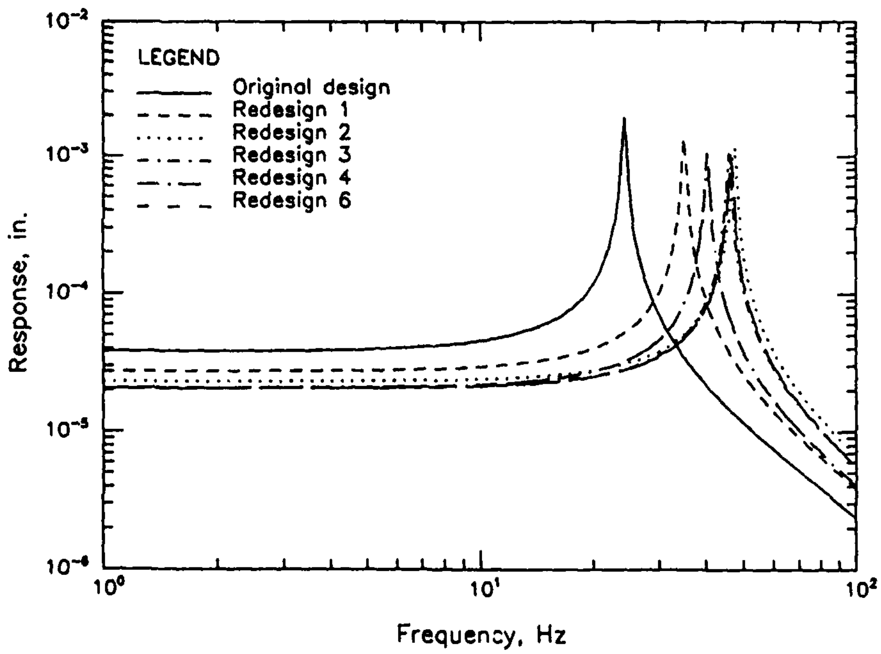

Figure 8. Box beam response history

8.2 Long Slender Truss: Frequency Response

The second problem is a long slender plane truss with 50 bays, each 12 inches square. The truss is fixed at one end and free at the other (Figure 9). The truss is loaded by a transverse force located at 80% of the distance from the root to the free end. The force is assumed to have uniform frequency content. The objective is to reduce the tip rotation with minimal weight increase. For design purposes, the truss is divided into five segments of equal length. Within each segment, two design variables are specified: one for the chord members and one for the diagonal members. For simplicity, all members are started with cross-sections of one square inch. Members are assumed to carry only axial force.

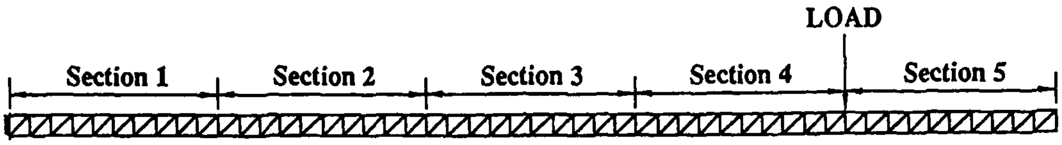

Figure 9. Space truss

Initially, the response is as shown in Figure 10. The goal is to reduce the peak responses for the first and third modes while maintaining the second mode peak at approximately the same value or less. The peak values and goals selected are as follows:

Mode	Initial Peak Response (μ-radians)	Desired Maximum Peak Response (μ-radians)
1	4735	1000
2	8.96	10.0
3	58.45	20.0

Technically, these "goals" are implemented as behavior constraints. Nominally, minimal weight is the objective, but the optimizer ignores the weight as long as any constraints are active. Instead, it merely tries to achieve a feasible design.

Figure 11 shows a history of the frequency response as the design evolves. The first mode peak is reduced to approximately 1000 micro-radians, the second peak is held at approximately its original value, and the third mode is reduced to a value slightly lower than the allowed maximum. The history of the individual peaks may be seen more clearly in Figure 12. Since the initial design violated two constraints by approximately a factor of five, the optimizer let the weight increase (by a factor of about three) during the initial iterations. Once the constraints were satisfied, it was able to reduce the weight slightly.

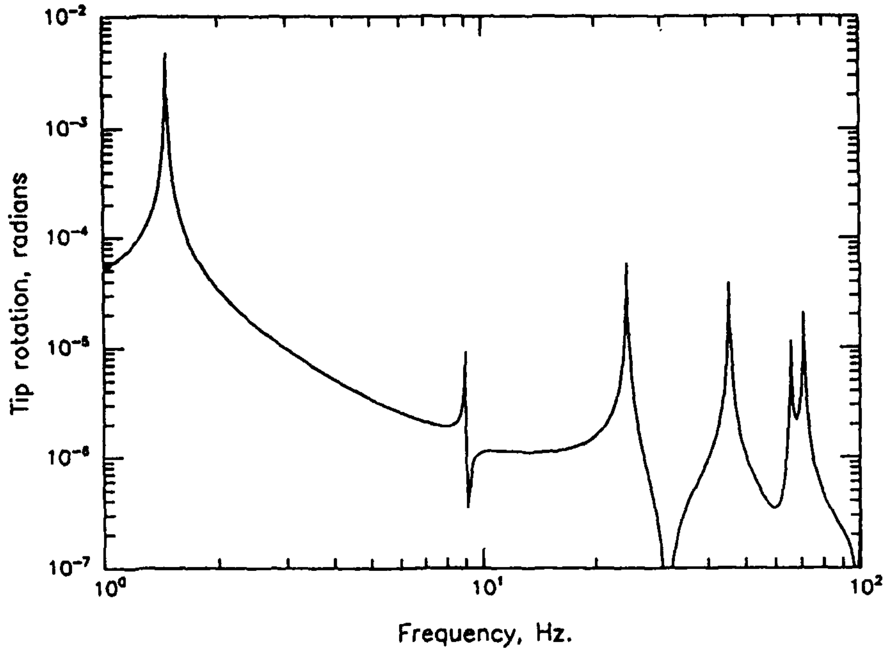

Figure 10. Truss optimization: initial frequency response

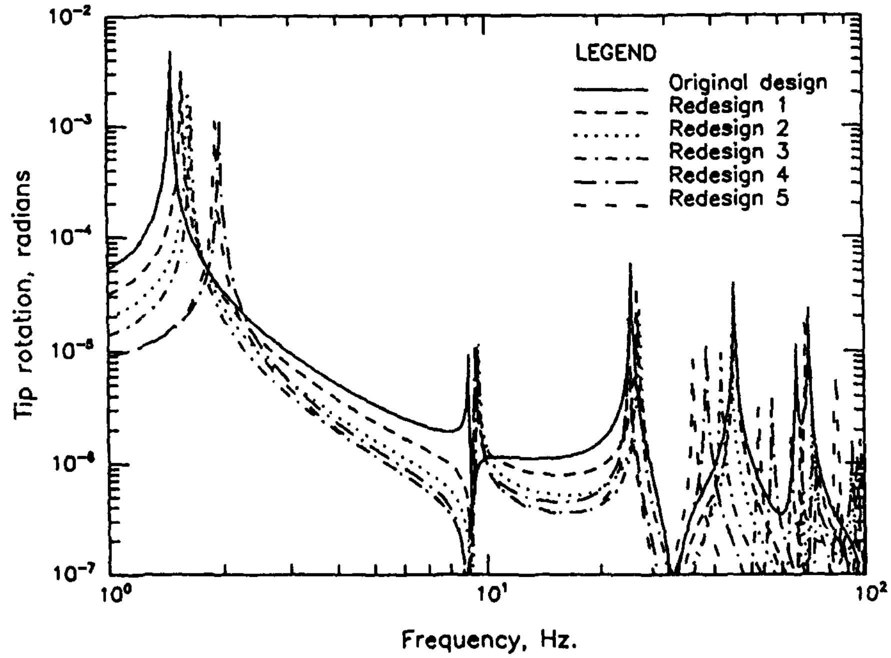

Figure 11. Truss frequency response design history

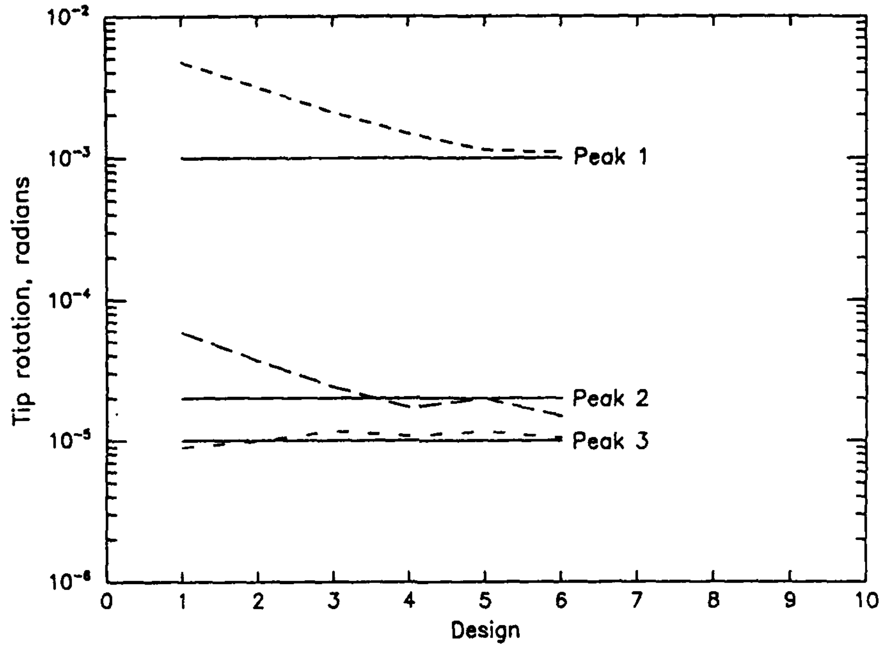

Figure 12. Truss response peak design history

8.3 Long Slender Truss: Transient Response

The same truss was used for optimization under transient dynamic loads. The truss was subject to a transient load whose sawtooth time record may be seen in Figure 13. A design model was prepared using only the chord members as design variables, with peaks at 0.1, 0.15, 0.2, and 0.25 sec not to exceed 0.3 in. Optimization was begun with an infeasible design, i.e., the peak values were greater than 0.3. Optimization then proceeded until all peaks were satisfied (Figure 14). The weight had increased initially, and then dropped off after the constraints were satisfied (Figure 15). Note that it is not necessary to specify the exact time at which each constrained peak occurs. A search is made in the neighborhood of each specified time for a local maximum, and then the maximum is tracked as the design evolves.

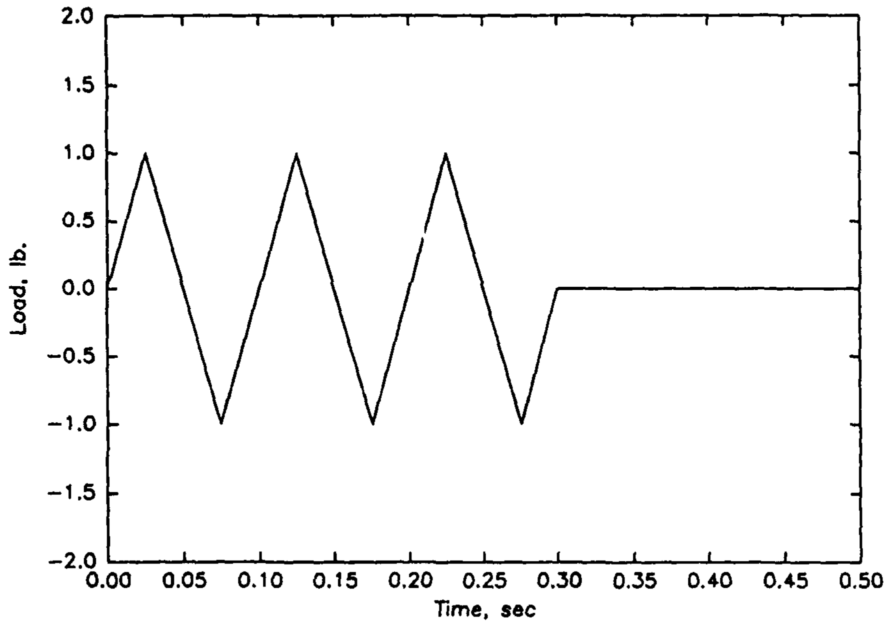

Figure 13. Load specified for transient optimization

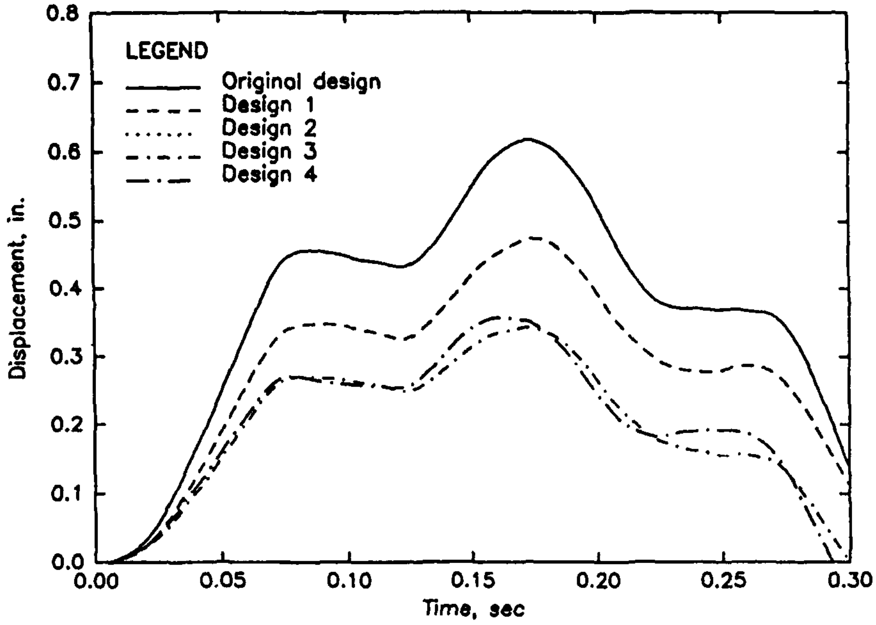

Figure 14. Truss transient response history

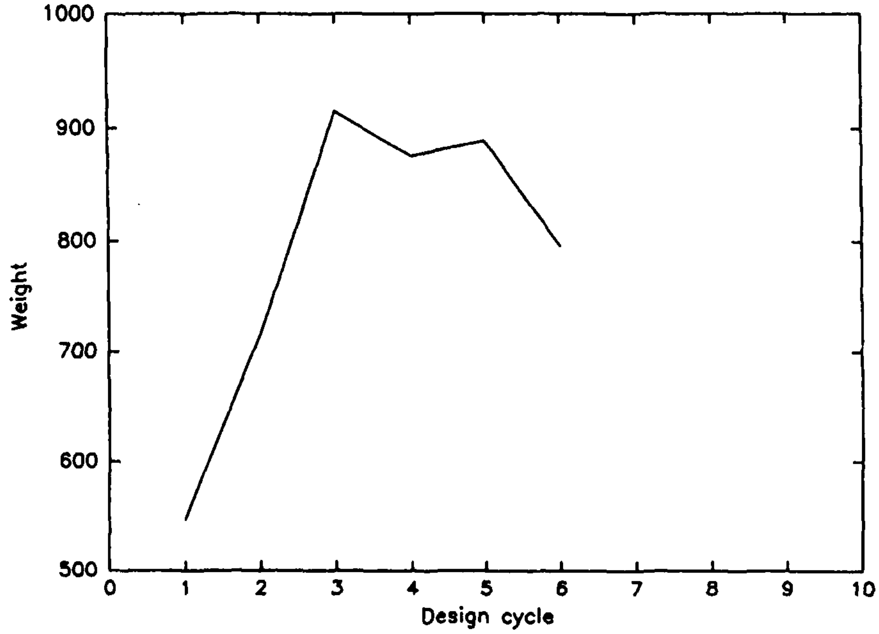

Figure 15. Truss transient weight history

8.4 Truss: Transient Response, Alternate Starting Design

The same problem was rerun, this time starting with a design having excessive weight, for which constraints were satisfied. This time the responses are as shown in Figures 16 and 17. Initially, the weight decreases drastically and the response increases until the constraint is reached at 0.3 in. Thereafter, the weight continues to decrease slightly while the constraint is satisfied (or nearly so). This particular starting point produced a lower weight than that reported in the previous section. However, the trend in that case was down and it was likely that more weight could have been shed if more optimization cycles had been performed.

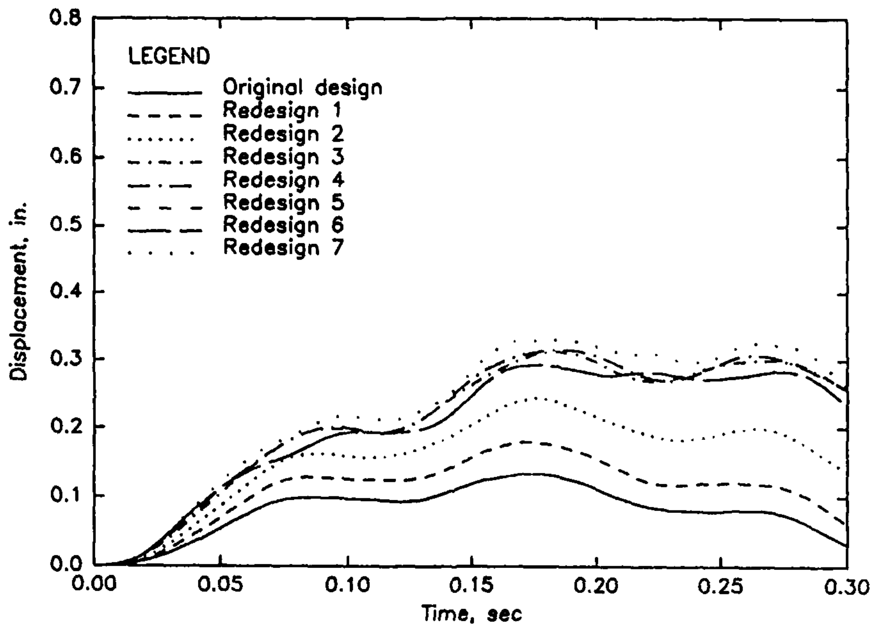

Figure 16. Truss transient response history, alternate starting design

8.5 Truss with Active Controller: Frequency Domain

The truss described in Section 8.2 was provided with active controllers on four of its diagonals (Figure 18).

These controllers were set up to sense velocity and provide reactive forces in the diagonals proportional to the sensed velocities. The gains were somewhat arbitrarily set to 400,000 lb/in/sec. The response of the truss to the same load was then as shown in Figure 19. As might be expected, the controllers were very effective at damping out higher modes but had virtually no impact on the lower mode. Therefore it was decided to let the optimizer tune the structure to consider only

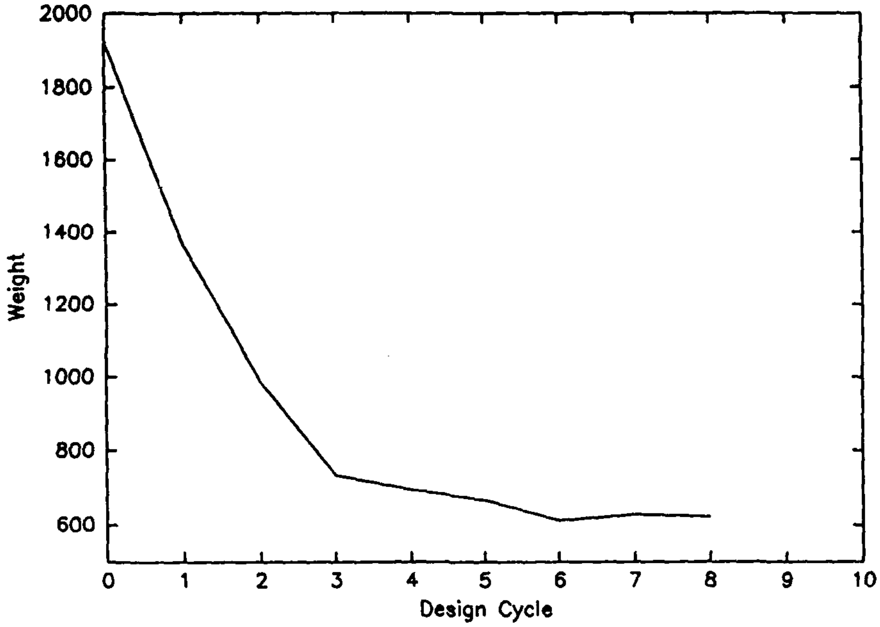

Figure 17. Truss transient weight history, alternate starting design

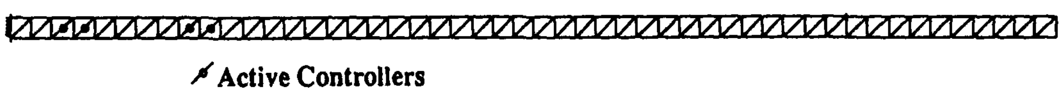

Figure 18. Truss with active controllers on selected diagonals

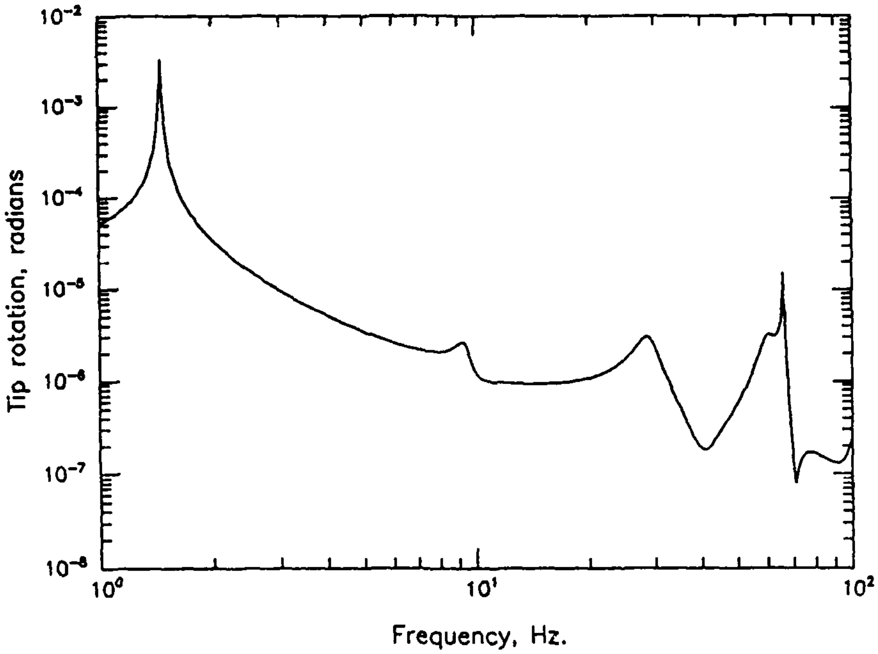

Figure 19. Truss with active controllers: initial response

the first peak, relying on the controllers to handle the other peaks. The frequency response history may be seen in Figure 20, and the first peak history in Figure 21. As the response history shows, the structural optimization defeated the damping provided by the controllers for the higher modes. This can be attributed to the fact that the optimizer reduced the members at which controllers were located to very small sizes. The effect of this change was to drive energy out of these members and reduce the effectiveness of the damping supplied by the controllers to these members. With hindsight, it is clear that controllers should have been placed at locations which were more effective in higher modes, and the optimizer should not have been allowed to reduce these members.

8.6 Summary

The examples shown here were chosen to illustrate the kinds of problems that DYNOPT can do. Optimization for either steady-state or transient loads was shown, and an example with a control system was run. However, the structures shown are not sufficiently complex or realistic to demonstrate the quantitative payoffs that can be expected on real-life structures Furthermore, it is a mistake to judge an optimization code by the amount of weight reduction that is demonstrated on any particular example. This is because the weight history can be skewed drastically by arbitrary selection of a very heavy starting design. DYNOPT should be judged by the potential shown in the forgoing examples, and by the payoffs that will be demonstrated on real structures in Phase II.

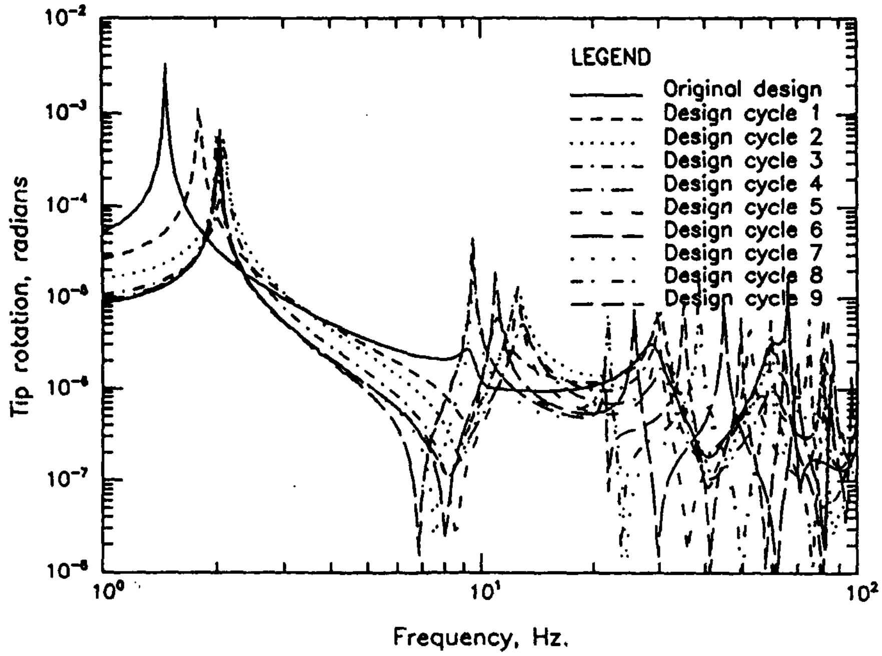

Figure 20. Truss with active controllers: response history

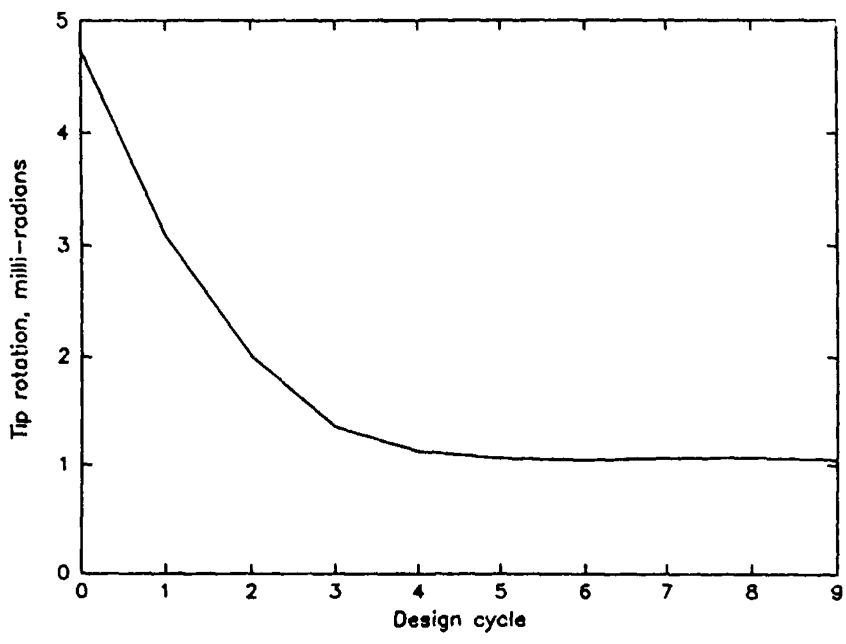

Figure 21. Truss with active controllers: first peak history

9. Link to a Viscoelastic Material Database

Under another Air Force contract, CSA developed an optimization code called ODAMP [13] that selects optimal damping treatments on the basis of modal strain energy. Under this effort, ODAMP has been made to interface with a database of viscoelastic material properties, also developed by CSA. The reason for this development is that the shear moduli and thicknesses selected by ODAMP may not be available in any actual materials. This new capability enables users to select actual materials that most nearly match the computer-generated optimal values. After selecting real materials, the user may wish to re-compute modal strain energies and perhaps perform more optimization cycles. Also, knowing loss factor values, he may wish to perform forced response calculations.

For a particular viscoelastic shear modulus design variable the user may request a search to find the material that most closely matches the optimal value found for that particular shear modulus. In order to define the search properly, the user must specify some additional data that is not considered in the optimization, including a temperature and a minimum acceptable loss factor. The user must also specify a range of frequencies, which would typically span the natural frequencies that were included in the optimization. Alternatively, the user may specify a particular frequency and give a range of modulus values.

The search software returns with a report of the number of materials which satisfy the specifications, if any. They are identified by name and identifying number. The user may then request evaluations of these materials for a range of temperatures and frequencies. This report returns values of shear modulus and loss factor for each pair of temperature and frequency values.

Figure 22 is a log of an interactive session using this capability. The initial SHOW command shows two VEM design variables, and the selected G values, as well as the natural frequencies of the modes whose modal strain energy was maximized. The RANGE,FREQUENCY command and the F_LOWER and F_UPPER commands specify the frequency range for the search. Temperature and minimum acceptable loss factor are specified, and a search is requested. At present, the database contains a limited number of materials, and consequently only a single material is found, "polyurethane 24-8-1." The user then asks for evaluation of the material. Modulus and loss factor are then displayed for a range of frequencies. The user then selects the other design variable, OUT_VEMG, for which the optimal value of G is 11,200 psi. After changing the loss factor cutoff, another search is conducted and two materials are found, including polyurethane 24-8-1 which also satisfied the previous search. The user would then presumably request a forced response analysis of his structure using the properties of this material.

```
ODAMP>SHOW
 Modes used in optimization:
   1      73.200
   2     101.000
   3     119.900
 Design variables used in optimization
   Name     Optimal G
    LEG_VEMG    9840.000
    OUT_VEMG   11220.000
ODAMP>SELECT,LEG_VEMG
 Optimal G for LEG_VEMG is  9840.00
ODAMP>RANGE,FREQ
ODAMP>F_LOWER=60
ODAMP>F_UPPER=200
ODAMP>TEMP=70
ODAMP>ETA_MIN=0.9
ODAMP>SEARCH
    1 materials were found.
  ID       Description
    2   POLYURETHANE 24-8-1
 Enter EVALUATE,id to evaluate a particular material
 Where "id" is chosen from the list above.
ODAMP>EVALUATE,2
 Material    Temperature   Frequency  Modulus   Loss factor
     2          70.00        60.00     8463.65      0.949
     2          70.00        67.00     8989.31      0.938
     2          70.00        74.00     9489.46      0.928
     2          70.00        81.00     9967.27      0.918
     2          70.00        88.00    10425.30      0.909
     2          70.00        95.00    10865.62      0.900
     2          70.00       102.00    11289.94      0.892
     2          70.00       109.00    11699.72      0.884
     2          70.00       116.00    12096.18      0.876
     2          70.00       123.00    12480.40      0.869
     2          70.00       130.00    12853.29      0.862
     2          70.00       137.00    13215.65      0.855
     2          70.00       144.00    13568.22      0.849
     2          70.00       151.00    13911.60      0.842
     2          70.00       158.00    14246.39      0.836
     2          70.00       165.00    14573.07      0.830
     2          70.00       172.00    14892.12      0.825
     2          70.00       179.00    15203.94      0.819
     2          70.00       186.00    15508.91      0.814
     2          70.00       193.00    15807.39      0.809
     2          70.00       200.00    16099.68      0.804
ODAMP>SELECT,OUT_VEMG
 Optimal G for OUT_VEMG is 11220.00
ODAMP>ETA_MIN=0.6
ODAMP>SEARCH
    2 materials were found.
  ID       Description
    1   SMRD-100F-90 -M870525
    2   POLYURETHANE 24-8-1
 Enter EVALUATE,id to evaluate a particular material
 Where "id" is chosen from the list above.
```

Figure 22. Log of an interactive database query session

10. Beam Cross-section Design

An approach to design of beam cross-sections has been developed and partially tested. The problem with optimization of beam finite elements is that their cross-sections are usually described by integrated properties (area, two moments of inertia, and a torsion constant), and these parameters are not very good for design. This is because it may not be possible to select a realistic cross-section configuration corresponding to the integrated property values selected by the optimizer. In fact, in many cases, the optimizer will increase the moment of inertia without limit, providing more stiffness, while reducing the area, and thus the weight, indefinitely. Even if constraints are introduced to prevent unreasonable ratios of area to moment of inertia, the designer is must still choose the details of the section, and insure that secondary design criteria such as local buckling allowables are satisfied.

The basic idea here is to develop a library of cross-sections (rectangular, hollow tube, z-shape, etc.). For each shape, the conventional elastic properties (area, moment of inertia, torsion constant) are coded in a subroutine in terms of the geometric parameters defining that shape. Sensitivities of each property with respect to each geometric parameter are also defined.

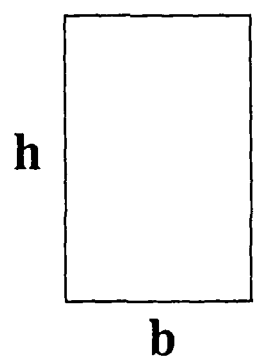

Figure 23. Rectangular cross-section

The rectangular cross-section (Figure 23) provides the simplest example. The geometric parameters are simply the height h and width b. The area A, bending moments of inertia I_1 and I_2, and the torsion constant K are coded in terms of h and b. Sensitivities ($\partial A/\partial h, \partial A/\partial b, \partial I_1/\partial h$, etc.) are also calculated. For the rectangular cross-section, the values and sensitivities of the slenderness ratios h/b and b/h are calculated. This information may be used to implement slenderness limits that can be used to prevent unreasonable shapes or avoid local buckling. The allowable stress for local buckling of a rectangular cross-section depends on the load in the beam, the length of the beam, and its material properties in addition to its cross-section parameters, i.e.,

$$\sigma \leq \sigma_{\max}(h, b, P, \text{length}, \text{material properties}) \tag{35}$$

In a redundant structure, the load internal P is an implicit function of the design variables. Therefore the total sensitivity of the local buckling constraint must include this implicit dependence, i.e.,

$$\left.\frac{\partial \sigma_{\max}}{\partial h}\right|_{\text{total}} = \frac{\partial \sigma_{\max}}{\partial h} + \frac{\partial \sigma_{\max}}{\partial P}\frac{\partial P}{\partial h} \tag{36}$$

These calculations would be done outside of the cross-section subroutines, however.

A symmetric Z-section was coded with four independent design variables (flange width and thickness; web depth and thickness – Figure 24). Two slenderness ratios (b_f/t_f and d_w/t_w) are defined.

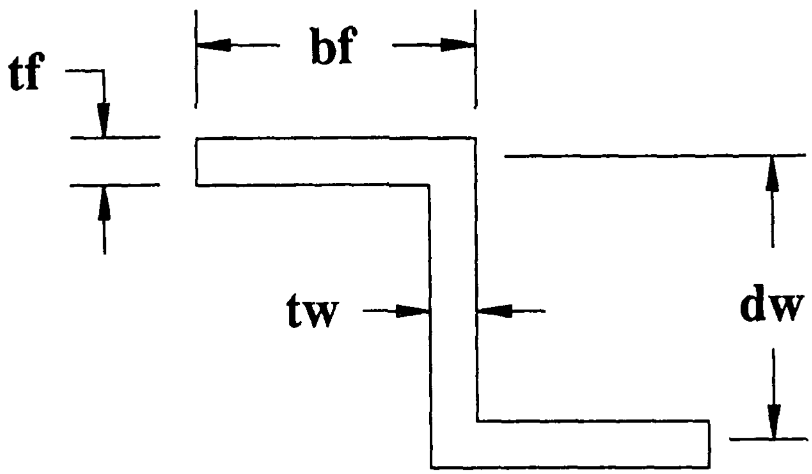

Figure 24. Z-shaped cross-section

Some rather simple tests were conducted to verify the sensitivity calculations, as shown in Figures 25 through 30. These were intended to provide visual confirmation of the tangent line based on the sensitivity, and to provide some measure of the nonlinearity of some of the integrated parameters with respect to particular design variables.

In addition to calculations of integrated properties and their sensitivities, it would be desirable to calculate extreme fiber stresses (and their sensitivities) given forces and moments (and their sensitivities). For the rectangular section, the

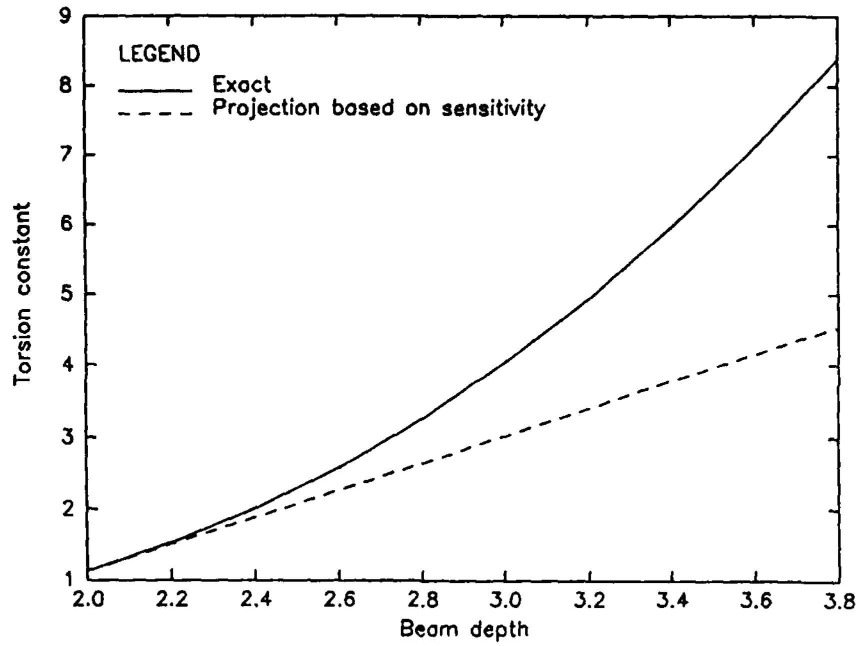

Figure 25. Rectangular section: torsion constant versus beam depth

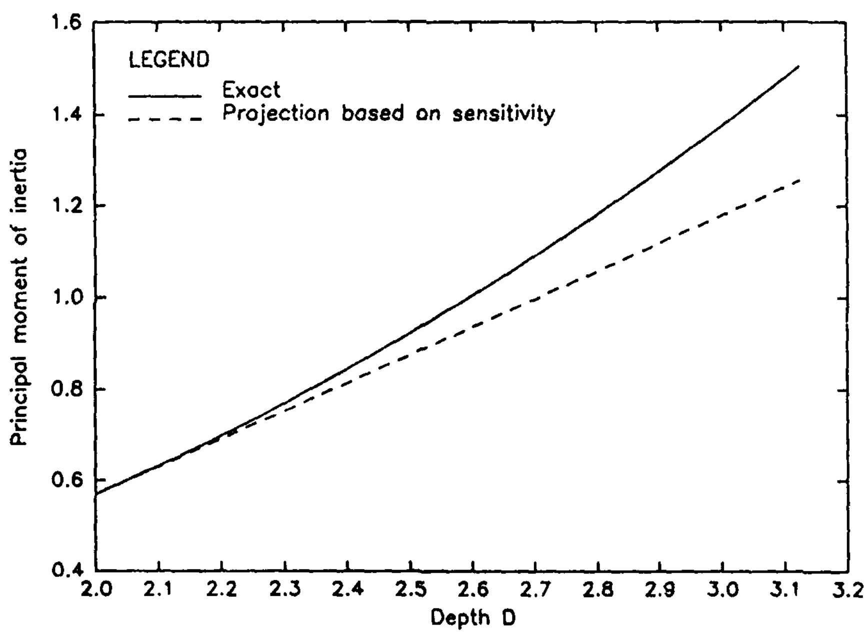

Figure 26. Z-section: moment of inertia versus depth

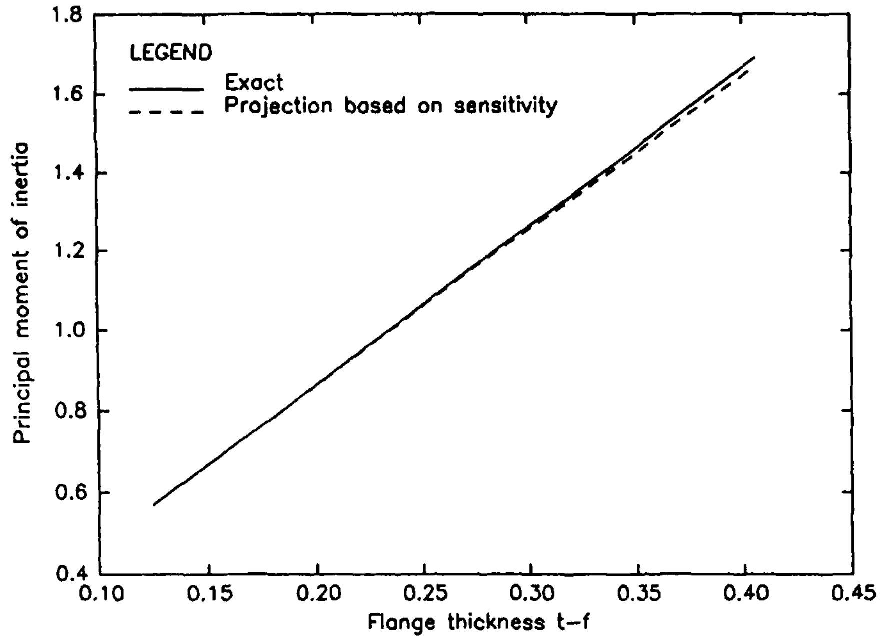

Figure 27. Z-section: moment of inertia versus flange thickness

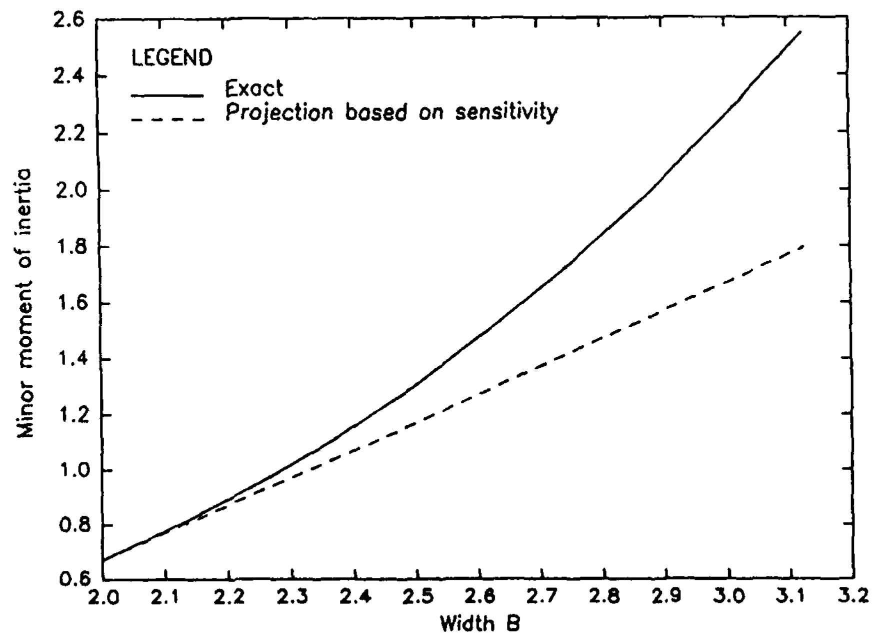

Figure 28. Z-section: moment of inertia versus width

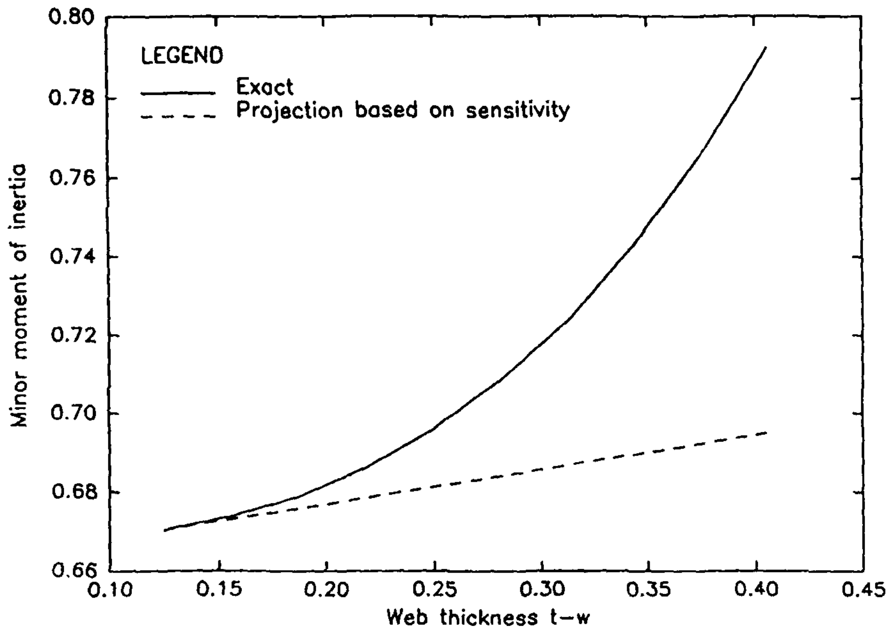

Figure 29. Z-section: moment of inertia versus web thickness

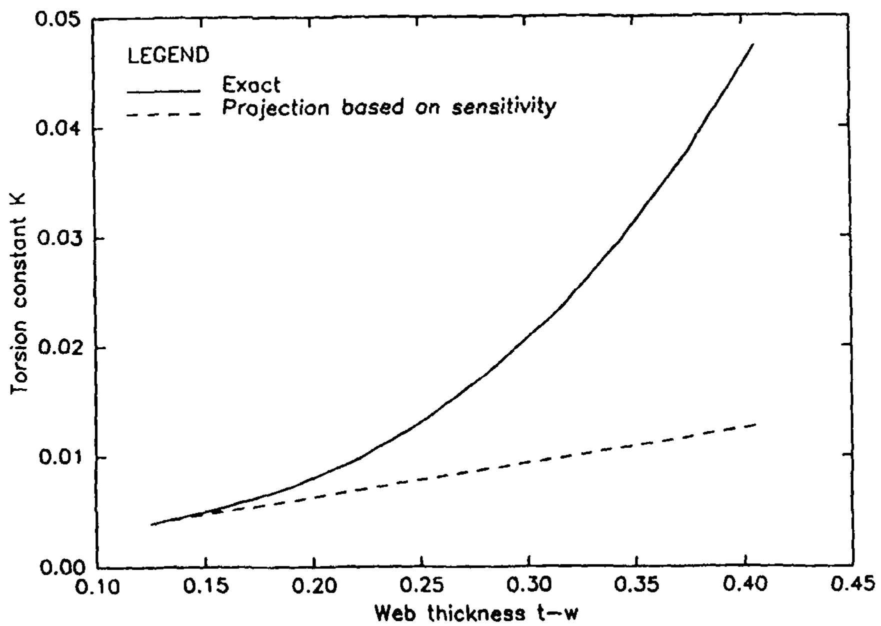

Figure 30. Z-section: torsion constant versus web thickness

stresses would be

$$\sigma = \frac{F}{A} \pm \frac{M}{S} \tag{37}$$

where S is the section modulus. Thus, for example,

$$\frac{\partial\sigma}{\partial h} = \frac{1}{A}\frac{\partial F}{\partial h} - \frac{F}{A^2}\frac{\partial A}{\partial h} \pm \frac{1}{S}\frac{\partial M}{\partial h} \mp \frac{M}{S^2}\frac{\partial S}{\partial h} \tag{38}$$

For other sections, it might be necessary to choose four or more points on the cross-section and code the relevant stress formulas for each point.

The above discussion assumes that all geometric parameters are retained as independent design variables in an optimization formulation, subject to constraints such as slenderness ratio limits. In some cases this could lead to an excessive number of independent design variables or stress constraints. In these cases it would be possible to provide for a reduced set of design variables. For example, one could allow only h to vary in a rectangular section, requiring a fixed h/b ratio. Such formulations could retain the same subroutine library with some additional overlaying code to carry out the reduction of some of the geometric parameters to dependent status.

Another point: a cross-section library could easily be augmented by a "generic" section in which properties such as section modulus are approximated by an empirical formula such as $S = k_1 A + k_2 I^{1/2}$.

The conclusion we draw from this study is that this "direct" approach to beam cross-sections appears quite promising. The advantages are as follows:

1. The designer can work directly with the geometric parameters that describe each cross-section, and does not have to worry about intermediate calculation of moments of inertia, etc.
2. Stresses and stress sensitivities can be recovered directly from forces and moments, and their sensitivities.
3. The coding involved for each cross-section is fairly simply. New cross-sections can be added as needed, or old ones modified.

As has been pointed out, a detailed description of each cross-section type may provide more freedom than is required for a particular application. There may, for example, be no unique combination of the four properties of a Z-section that satisfies a given design problem. There must always be provision for reducing the number of independent variables as mentioned above.

10.1 Summary

The examples shown here were chosen to illustrate the kinds of problems that DYNOPT can do. Optimization for either steady-state or transient loads was shown, and an example with a control system was run. However, the structures shown are not sufficientlv complex or realistic to demonstrate the quantitative payoffs that can be expected on real-life structures that are so much more complex than simple trusses and box beams. Furthermore, it is a mistake to judge an optimization code by the amount of weight reduction that is demonstrated on any particular example. This is because the weight history can be skewed drastically by arbitrary selection of a very heavy starting design. DYNOPT should be judged by the potential shown in the forgoing examples, and by the payoffs that will be demonstrated on real structures in Phase II.

11. Plans for Follow-on Software Development

There is a need for optimization software tailored to SDI structures. As was pointed out in the introduction, there are three fundamental approaches to vibration suppression for SDI structures: basic structural design, design of damping treatments, and control system design. This report has presented developments in optimization that support these design problems. They are shown in Figure 1 in relation to preceding developments in optimization. Phase II will expand the new optimization capabilities and integrate them with these previous capabilities in a new software package.

Phase II will consist of two basic parts: further testing and refinement of individual optimization techniques, and development of an integrated package. Testing and refinement is needed to shake out software shortcomings and any difficulties that may arise with large problems. Integration is needed to make it possible to exploit each technology where it is most effective.

The following criteria are proposed for the software:

1. It should support constraints, objectives, and design variables that are important for SDI structures. These include the following:

   (a) Design variables: Member sizes, gage thicknesses, damping material properties and thicknesses, other material properties, and concentrated spring and mass properties.

   (b) Constraints: Static stiffness and stress, natural frequencies and mode shapes, modal strain energy, dynamic displacements and stresses.

   (c) Objective: weight, or any response that can be constrained.

   Steps should be taken to move away from dependence on MSC/NASTRAN so that analysis results and sensitivities could eventually be obtained from other codes such as ASTROS and COSMIC NASTRAN. This could be done by obtaining this information from a neutral database rather than directly from MSC/NASTRAN. This development would also facilitate calculation of sensitivities using existing codes that do not provide explicit sensitivity calculations [14].

2. It should use existing software where possible. CADDB [15] is a likely database software candidate. CADDB provides the database for the Air Force's ASTROS code [16], and is specially designed for finite element or other engineering data. CADDB may be accessed through calls from MAPOL, ASTROS' high-level language, through Fortran calls, or interactively via the ICE [17] interface, using an SQL-like query language.

3. It should provide for unattended computation of optimal solutions, or for user intervention at any stage. User interaction should be made via modern screen management software and the associated database software.
4. It should provide for graphic displays of optimization data such as design history or response history. The link to display software should be such that design trends may be displayed graphically on three-dimensional structure plots (e.g., in PATRAN).

ASTROS is an excellent optimization code. However, it is tailored to aircraft structures and is thus not entirely suitable for SDI structures. In addition to CADDB, much could be gained by studying the ASTROS code and perhaps borrowing pieces of it.

These points will be elaborated in the Phase II proposal for this SBIR effort.

Appendix
Eigenvalue and Eigenvector Sensitivity Equations

Following are derivations of eigenvalue and eigenvector sensitivity equations. Assume a particular mode has been chosen. The eigenvalue equation is

$$[\mathbf{K} - \lambda \mathbf{M}]\mathbf{\Phi} = 0 \tag{A.1}$$

Differentiate:

$$[\mathbf{K}' - \lambda' \mathbf{M} - \lambda \mathbf{M}']\mathbf{\Phi} + [\mathbf{K} - \lambda \mathbf{M}]\mathbf{\Phi}' = 0 \tag{A.2}$$

Premultiplying by $\mathbf{\Phi}^T$,

$$\mathbf{\Phi}^T[\mathbf{K}' - \lambda' \mathbf{M} - \lambda \mathbf{M}']\mathbf{\Phi} + \mathbf{\Phi}^T[\mathbf{K} - \lambda \mathbf{M}]\mathbf{\Phi}' = 0 \tag{A.3}$$

From transposing (A.1) we know that $\mathbf{\Phi}^T\mathbf{K} = \lambda \mathbf{\Phi}^T \mathbf{M}$, so that the second term of (A.3) drops out, and, solving for λ',

$$\lambda' = \frac{\mathbf{\Phi}^T(\mathbf{K}' - \lambda \mathbf{M}')\mathbf{\Phi}}{\mathbf{\Phi}^T \mathbf{M} \mathbf{\Phi}} \tag{A.4}$$

Note that the denominator is simply the modal mass, and eigenvectors are usually normalized to give this quantity unit value.

For eigenvector sensitivity, rewrite Equation (A.2) as:

$$\mathbf{D}\mathbf{\Phi}' = \mathbf{f} \tag{A.5}$$

where

$$\mathbf{D} = \mathbf{K} - \lambda \mathbf{M} \tag{A.6}$$

and

$$\mathbf{f} = [\lambda' \mathbf{M} + \lambda \mathbf{M}' - \mathbf{K}]\mathbf{\Phi} \tag{A.7}$$

Equation (A.5) is singular but **D** may be reduced by one order by invoking the normalizing equation for $\mathbf{\Phi}$. [1] It is then solved like a static problem with one right-hand side per design variable. **D** retains the sparseness of the original mass and stiffness matrices. The whole process must be repeated for each mode to be differentiated. This process can be quite time-consuming, depending on the number of mode shapes to be differentiated and the number of design variables.

The ingredients of both eigenvalue and eigenvector sensitivities are the stiffness and mass derivatives which may be obtained by running the DSVGx modules in MSC/NASTRAN or by a finite difference operation.

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[1] In the case of repeated roots of multiplicity m, special steps must be taken to reduce the order of **D** by m. This case is not dealt with here.

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